Sub-Riemannian PDEs, minima of functionals and application to brain modelling

Research project

The project revolves around several aspects of geometric analysis, including but not limited to: geometric analysis of sub-Riemannian spaces, Elliptic, sub-Elliptic, parabolic PDEs, geometric flows and calculus of variations, applications to vision and brain modelling. The scope is to develop one or more of following research topics, which are some of the main objectives of the project PRIN 2022_CITTI, "Regularity problems in sub-Riemannian structures" 2022F4F2LH - CUP J53D23003760006

1 **Geometric analysis of sub-Riemannian spaces** A Sub-Riemannian (SR) structure is a regular manifold with a sub-bundle of the tangent bundle, called horizontal space, and a metric on it. Local generators of this sub-bundle, called horizontal vector fields, encode the directions of propagation of mechanical, physical, biological or geometrical phenomena. All the relevant differential objects of a SR manifold are expressed in terms of these vector-fields. The most challenging problems still open in the area of SR structures are properties of manifolds and geodesics.

2. Sub-Elliptic PDEs, parabolic PDEs, geometric flows and calculus of variations The interior regularity for linear degenerate PDEs of sub-elliptic type is a very well-understood problem and one of the main tools is the celebrated lifting and freezing technique due to Rothschild and Stein. However, regularity at the boundary or regularity in non linear sub-Elliptic and parabolic PDEs, are still open problems. We are particularly intersted in geometric flows, p-Laplacian type equations, or equations for differential forms in SR manifolds.

An other class of challening problems comes from regularity of vector-valued minimizers of energy integrals of calculus of variations, in elliptic and sub-elliptic setting.

3. Applications to vision and brain modelling Tools from analysis and PDEs can be used to formalize models of the human brain. Starting from the work by Hubel-Wiesel, several authors proposed SR

models for the neural activity in the primary visual cortex, able to explain visual perception phenomena. We plan to develop models of the motor cortex with the same tools.

Integro-differential Equations can be used to model propagation of degenerative diseases such as Alzheimer's disease (AD). In this project we aim to develop a mathematical model for the interplay of proteins Abeta and tau and the progression of AD, which play a central role in the onset and spreading of AD.

Activity plan

The candidates are expected to have a proven, previous exposure, and possibly research experience, in at least one of the previous described research topic.

The selected fellow will work on one or more of the research topics listed above. He/she collaborate with one or more of the members of the local unit of the project PRIN 2022_CITTI 2022F4F2LH - CUP J53D23003760006. The research group is composed by Giovanna Citti, Annalisa Baldi, Andrea Bonfiglioli, Giovanni Cupini, Annamaria Montanari, Daniele Morbidelli, Maria Carla Tesi.